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★**Dynamics and bifurcations.**

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An unusual book in both content and style, as the authors rightly describe it, this text offers an introduction to dynamical systems through careful and systematic study of low-dimensional differential equations and maps for which fundamental ideas can be explained without too much obscuring formalism.

The account proceeds step by step through increasing dimension, beginning (dimension 1) with scalar autonomous ordinary differential equations and their elementary bifurcations, scalar maps including the logistic map, then (dimension $1\frac{1}{2}$) nonautonomous scalar systems. The majority of the book is devoted to planar autonomous ODEs (dimension 2) from which many important ideas and methods are extracted, including local linearization, stability, bifurcation of equilibria (comparing Lyapunov-Schmidt and center manifold techniques), periodic orbits and global attractors. The full richness of the subject begins to emerge with planar maps, where, after basic linear theory, the subjects briefly discussed include stable and unstable manifolds and homoclinic orbits as well as saddle-node, period-doubling and Poincare-Andronov-Hopf bifurcations. There is an instructive short section on hazards of discretization, with a startling appearance of the Henon map in a difference scheme for numerically integrating the harmless logistic differential equation.

Topics touched upon in higher dimensions include forced oscillations (dimension $2\frac{1}{2}$), bifurcation to invariant tori (dimension 3) and Hamiltonian systems with two degrees of freedom (dimension 4). Key notions are explained with clear illustrations, although many of the proofs are beyond the scope of this undergraduate or beginning graduate text. Each chapter ends with a discussion of relevant literature, and a catalogue of background theorems from analysis is given in an appendix.

A major feature of the book is the use made of the PHASER software for plotting orbits of differential equations and maps. This computer program accompanies Kocak's earlier book [89k:58003Differential and difference equations through computer experiments, second edition, Springer, New York, 1989; MR 89k:58003], and supports the authors' philosophy that in dynamical systems, as in most parts of mathematics, it is through grappling with specific

examples that general theoretical principles are best understood. The tools and examples provided by PHASER are most effectively used to illustrate the text and provide a vehicle for practical investigations among the numerous well-motivated and interesting exercises.

This book stands out from many other textbooks by its gentle explanatory style and polite consideration for the reader, emphasized by the pleasing decorations at chapter ends and elsewhere by Halil Buttanri.   *D. R. J. Chillingworth* (4-SHMP)